Government of **Western Australia**
Curriculum Council

WACE
Western Australian Certificate of Education

MATHEMATICS: SPECIALIST

3C/3D
Calculator-assumed

WACE Examination 2010

Final Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

Section One: Calculator-assumed (80 Marks)

Question 8 (4 marks)

Points P and Q have coordinates $(3,\ 1,\ -2)$ and $(4,\ 2,\ -1)$ respectively.

(a) Write a vector equation for the line passing through P and Q. (2 marks)

Solution
$\overrightarrow{PQ} = (4\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}) - (3\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}) = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ $\boldsymbol{r} = 3\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k} + \lambda(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ or $\boldsymbol{r} = 4\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k} + \lambda(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$
Specific behaviours
✓ finds the direction vector of the line ✓ states a vector equation of the line

(b) Show that the vector $2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$ is perpendicular to the line through P and Q. (1 mark)

Solution
The line through P and Q has direction vector $\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $(2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}).(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) = 2 - 1 - 1 = 0$ The two vectors are perpendicular
Specific behaviours
✓ shows that the dot product $(2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}).(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ is zero

(c) Write down a vector equation of the plane containing P and Q with $2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$ as its normal vector. (1 mark)

Solution
$\boldsymbol{r} \cdot (2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}) = (3\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}) \cdot (2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k})$ or $\boldsymbol{r} \cdot (2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}) = (4\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}) \cdot (2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k})$ i.e. $\boldsymbol{r} \cdot (2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}) = 7$
Specific behaviours
✓ states a vector equation of the plane

Question 9 **(5 marks)**

The vertices of the triangle ABC have coordinates $A(2, 1, 4)$, $B(7, 1, 0)$ and $C(-7, 5, 2)$.

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. (2 marks)

Solution
$\overrightarrow{AB} = 5\boldsymbol{i} - 4\boldsymbol{k}$ and $\overrightarrow{AC} = -9\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$
Specific behaviours
✓✓ finds both vectors

(b) Find the size of $\angle BAC$, correct to the nearest degree. (1 mark)

Solution
$\overrightarrow{AB} \cdot \overrightarrow{AC} = \lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{AC}\rvert \cos BAC$ i.e. $-37 = \sqrt{41} \times \sqrt{101} \cos BAC$ i.e. $\angle BAC = 125.01^{o} = 125^{o}$ **Or:** Use a CAS: angle([5, 0, –4],[–9, 4, –2]) = 125.10º = 125º
Specific behaviours
✓ correctly solves for $\angle BAC$

(c) The point D divides $\overrightarrow{AC}$ internally in the ratio 5:1. Find the vector $\overrightarrow{BD}$. (2 marks)

Solution
$\overrightarrow{BD} = \overrightarrow{BA} + \frac{5}{6}\overrightarrow{AC}$ i.e. $\overrightarrow{BD} = -5\boldsymbol{i} + 4\boldsymbol{k} + \frac{5}{6}(-9\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k})$ i.e. $\overrightarrow{BD} = -\frac{25}{2}\boldsymbol{i} + \frac{10}{3}\boldsymbol{j} + \frac{7}{3}\boldsymbol{k}$
Specific behaviours
✓ correctly expresses $\overrightarrow{BD}$ in terms of $\overrightarrow{BA}$ and $\overrightarrow{AC}$ ✓ finds $\overrightarrow{BD}$

Question 10 **(7 marks)**

The function $y = f(x)$ is given by $f(x) = e^{ax-2}$, where a is a constant.

When $y = f(x)$ and $y = f^{-1}(x)$ are plotted on the same set of axes, they intersect at a point where $x = 3$.

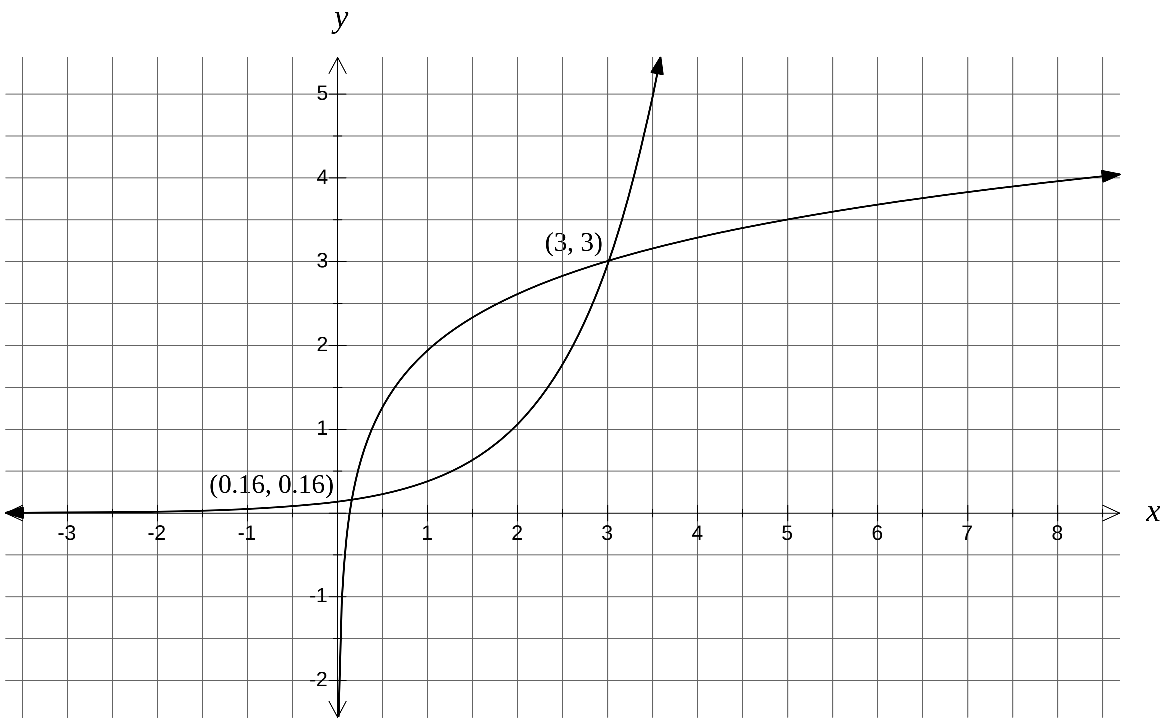
(a) Write down the exact value of y at this point of intersection. (1 mark)

Solution
$y = 3$
Specific behaviours
✓ recognises that a function and its inverse intersect on the line $y = x$

(b) Find the value of a correct to 2 decimal places. (2 marks)

Solution
Point $(3,\ 3)$ lies on the graph of $y = f(x)$ i.e. $e^{3a-2} = 3$ Solve using a CAS i.e. $a = 1.03$
Specific behaviours
✓ sets up the equation using the point $(3,\ 3)$ ✓ solves for a

(c) Draw $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes, showing the coordinates of the point(s) of intersection. (4 marks)

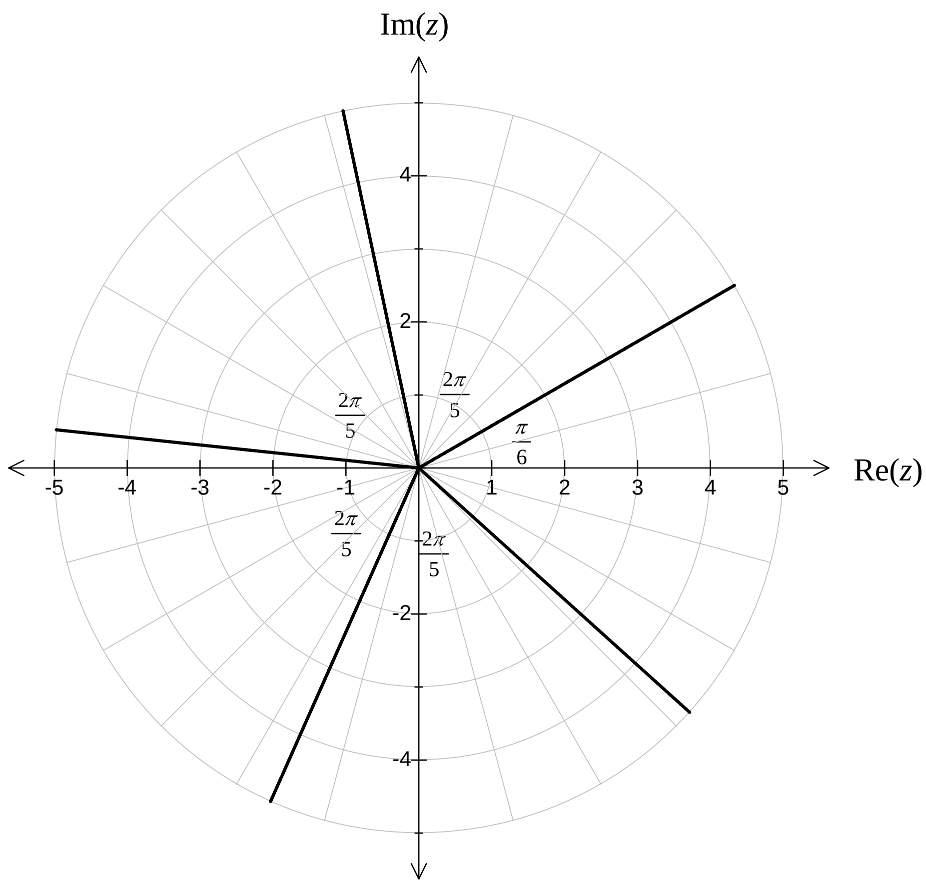
Solution

Specific behaviours
✓ sketches the graph of $y = f(x)$ **And either:** ✓ uses a CAS to determine $y = f^{-1}(x) = \dfrac{\ln x + 2}{1.03}$ ✓ sketches the graph of $y = f^{-1}(x)$ ✓ identifies the point of intersection $(0.16,\ 0.16)$. Note: $(3,3)$ previously identified **Or**: ✓✓sketches the graph of $y = f^{-1}(x)$ as a reflection of $y = f(x)$, with correct asymptote and correct shape ✓ identifies the point of intersection $(0.16,\ 0.16)$. Note: $(3,3)$ previously identified

Question 11 **(9 marks)**

(a) Using your CAS calculator (or otherwise) find all solutions to $z^5 = \frac{3125}{2}\left(-\sqrt{3}+i\right)$ in exact polar form, where $z = r\left(\cos\theta + i\sin\theta\right)$, $-\pi < \theta \le \pi$ and $r \ge 0$. (5 marks)

Solution
On CAS: cexpand$\left(\left(-\frac{3125\sqrt{3}}{2} + \frac{3125i}{2}\right)^{1/5}\right) = \frac{5\sqrt{3}}{2} + \frac{5i}{2}$ compToTrig$\left(\frac{5\sqrt{3}}{2} + \frac{5i}{2}\right) = 5\left(\cos\frac{\pi}{6} + \sin\frac{\pi}{6}i\right)$ $z_1 = 5\left(\cos\frac{\pi}{6} + \sin\frac{\pi}{6}i\right)$ Roots are equally spaced and so are $\frac{2\pi}{5}$ radians apart. $z_2 = 5\left(\cos\frac{17\pi}{30} + \sin\frac{17\pi}{30}i\right)$ $z_3 = 5\left(\cos\frac{29\pi}{30} + \sin\frac{29\pi}{30}i\right)$ $z_4 = 5\left(\cos\left(\frac{-7\pi}{30}\right) + \sin\left(\frac{-7\pi}{30}\right)i\right)$ $z_5 = 5\left(\cos\left(\frac{-19\pi}{30}\right) + \sin\left(\frac{-19\pi}{30}\right)i\right)$
Specific behaviours
✓ uses a calculator to find one root of the equation in Cartesian form ✓ uses a calculator to convert this root from Cartesian to polar form ✓ recognises that the roots are $\frac{2\pi}{5}$ radians apart ✓✓finds the remaining 4 roots

(b) Draw the solutions from (a) on the complex plane below. Show all major features.
(4 marks)

Solution


Specific behaviours
✓ shows that each root has magnitude 5
✓ shows that the first root has amplitude $\frac{\pi}{6}$
✓✓accurately spaces the other roots at intervals of $\frac{2\pi}{5}$ radians

Question 12 **(6 marks)**

(a) Use the method of mathematical induction to prove that

$1^2+2^2+3^2+\ldots+n^2=\frac{1}{6}n(n+1)(2n+1)$ for $n\geq 1$. (5 marks)

Solution
Let $P(n)$ be the statement $1^2+2^2+3^2+\ldots+n^2=\frac{1}{6}n(n+1)(2n+1)$ $P(1)$ is true because $\frac{1}{6}1(1+1)(2\times1+1)=1=1^2$ Assume $P(k)$ is true. i.e. assume $1^2+2^2+3^2+\ldots+k^2=\frac{1}{6}k(k+1)(2k+1)$ Consider $P(k+1)$. Required to deduce that $1^2+2^2+3^2+\ldots+k^2+(k+1)^2=\frac{1}{6}(k+1)(k+2)(2k+3)$ L.S. $=1^2+2^2+3^2+\ldots+k^2+(k+1)^2$ $=\frac{1}{6}k(k+1)(2k+1)+(k+1)^2$ (using the assumption) $=\frac{1}{6}(k+1)(k(2k+1)+6(k+1))$ $=\frac{1}{6}(k+1)(2k^2+7k+6)$ $=\frac{1}{6}(k+1)(k+2)(2k+3)$ = R.S. Thus $P(k)\Rightarrow P(k+1)$ and $P(1)$ is true. Hence result
Specific behaviours
✓ shows $P(1)$ is true ✓ accurately states the induction assumption ✓ substitutes $\frac{1}{6}k(k+1)(2k+1)$ for $1^2+2^2+3^2+\ldots+k^2$ in $1^2+2^2+3^2+\ldots+k^2+(k+1)^2$ ✓ correctly simplifies and factorises $\frac{1}{6}k(k+1)(2k+1)+(k+1)^2$ ✓ makes a final statement

(a) Hence evaluate $\lim_{n\to\infty}\left[\frac{1^2+2^2+3^2+\ldots+n^2}{n^3}\right]$. (1 mark)

Solution
$\lim_{n\to\infty}\left[\frac{1^2+2^2+3^2+\ldots+n^2}{n^3}\right]=\frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)=\frac{1}{3}$
Specific behaviours
✓ evaluates the limit

Question 13 **(9 marks)**

Bharat and Lena have been monitoring a small population of woylies introduced into a wildlife sanctuary. They have collected the following data on the females in the population:

Age (years)	0–1	1–2	2–3	3–4	4–5
Initial population	2	5	4	4	0
Birth rate	0	1.9	2.2	1.2	0.9
Survival rate	0.5	0.8	0.5	0.3	0

(a) Lena will now represent the above data as a Leslie Matrix to predict future populations. Write down such a matrix. (1 mark)

Solution

$$L = \begin{bmatrix} 0 & 1.9 & 2.2 & 1.2 & 0.9 \\ 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 \end{bmatrix}$$

Specific behaviours

✓ correctly derives the Leslie Matrix

(b) Showing working to support your answer, find (to the nearest whole number) the total female population after:

(i) 1 year

(ii) 2 years

(iii) 5 years (3 marks)

Solution

(i) 1 year

$= \begin{bmatrix}1 & 1 & 1 & 1 & 1\end{bmatrix} \times L \times P$

= 31

(ii) 2 years

$= \begin{bmatrix}1 & 1 & 1 & 1 & 1\end{bmatrix} \times L^2 \times P$

= 29

(iii) 5 years

$= \begin{bmatrix}1 & 1 & 1 & 1 & 1\end{bmatrix} \times L^5 \times P$

= 76

where

Leslie Matrix = L = $\begin{bmatrix}0 & 1.9 & 2.2 & 1.2 & 0.9 \\ 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0\end{bmatrix}$ Initial population = P = $\begin{bmatrix}2 \\ 5 \\ 4 \\ 4 \\ 0\end{bmatrix}$

Specific behaviours

✓✓✓ correctly calculates each population

(c) Lena must predict when the female population will be at least 10 times its original size. How soon should this happen? Justify your answer. (2 marks)

Solution

Initial population is 15

$\begin{bmatrix}1 & 1 & 1 & 1 & 1\end{bmatrix} \times L^7 \times P = 136$ $\quad \begin{bmatrix}1 & 1 & 1 & 1 & 1\end{bmatrix} \times L^8 \times P = 181$

Hence after 8 years

Specific behaviours

✓ states the initial population

✓ shows working to support the answer of 8 years

(d) After eight years, Bharat observes that the population is badly affected by disease and that the survival rates have decreased. The rate for 4–5 year-olds remains at 0 but all other rates have fallen to 0.2. If no action is taken, in which year will Bharat find that the female population has fallen below 100 again? Justify your answer. (3 marks)

Solution

New Leslie matrix = K = $\begin{bmatrix} 0 & 1.9 & 2.2 & 1.2 & 0.9 \\ 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 \end{bmatrix}$

Population in Year 9 = $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \times K \times L^8 \times P$ = 177

Population in Year 10 = $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \times K^2 \times L^8 \times P$ = 100

Thus the population falls below 100 in the 11th year.

Specific behaviours

- ✓ correctly states the new Leslie Matrix
- ✓ correctly calculates the woylie population for years 9 and 10
- ✓ identifies the year in which the population falls below 100

Question 14 **(10 marks)**

A ball oscillates on the end of a spring such that its vertical motion can be described by the solution of the differential equation, $\frac{d^2x}{dt^2} = -4x$, where $x(t)$ cm is the vertical displacement of the ball about the mean position and t is measured in seconds.

If the amplitude of motion is 4 cm and when $t = 0$, $x = 0$ and $\frac{dx}{dt} > 0$:

(a) (i) Determine $x(t)$. (2 marks)

Solution
$x = 4\sin 2t$
Specific behaviours
✓✓ correctly expresses x in terms of t

(ii) Find, exactly, the first two times $t > 0$ that the ball is 2 cm away from its mean position. (2 marks)

Solution
When $x = 2$, $4\sin 2t = 2$ Solve to give $x = \frac{\pi}{12}$ (first occasion) and $x = \frac{5\pi}{12}$ (second occasion)
Specific behaviours
✓✓correctly identifies the required times

(b) For what fraction of time will the ball be at least 2 cm away from its mean position? (3 marks)

Solution
The ball will be at least 2 cm away from its mean position for 2 intervals within a single period. Each interval lasts $\left(\frac{5\pi}{12} - \frac{\pi}{12}\right) = \frac{\pi}{3}$ seconds. The period of motion is π seconds. The required fraction is $\frac{2}{3}$.
Specific behaviours
✓ correctly identifies the length of each interval ✓ correctly identifies the length of one period ✓ correctly calculates the required fraction

(c) How far does the ball travel in the first 10 seconds? (3 marks)

Solution
In $3\pi = 9.425$ seconds, the ball will complete 3 cycles i.e. the ball will travel $3 \times 16 = 48$ cm In the remaining 0.575 seconds, the ball will travel $4\sin(2 \times 0.575) = 3.652$ cm Total distance = 51.652 cm
Specific behaviours
✓ correctly calculates how many complete cycles in 10 seconds ✓ correctly calculates the additional distance travelled ✓ calculates the final answer

Or:

Solution
Distance = $\int_0^{10} \left\| \frac{d}{dt}(4\sin 2t) \right\| dt$ i.e. D = $\int_0^{10} \|8\cos 2t\| dt$ i.e. D = 51.652 cm
Specific behaviours
✓ uses the formula: Distance = $\int_a^b \|v(t)\| dt$ with $a = 0,\ b = 10$ ✓ correctly determines the velocity of the ball ✓ uses a calculator to find the final answer

Question 15 **(9 marks)**

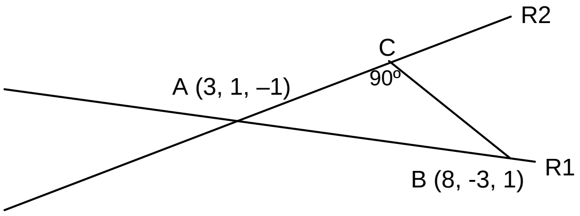
Two rockets are fired from different positions at the same time. Rocket 1 leaves from position $-7\boldsymbol{i}+9\boldsymbol{j}-5\boldsymbol{k}$ km at a velocity of $5\boldsymbol{i}-4\boldsymbol{j}+2\boldsymbol{k}$ km/min and Rocket 2 leaves from position $-6\boldsymbol{i}-5\boldsymbol{j}+2\boldsymbol{k}$ km at a velocity of $9\boldsymbol{i}+6\boldsymbol{j}-3\boldsymbol{k}$ km/min. Each rocket leaves a trail of smoke and, although the rockets do not collide, their smoke trails do intersect.

(a) Find the coordinates of the point at which the smoke trails intersect. (4 marks)

Solution
$\boldsymbol{r}_1 = -7\boldsymbol{i}+9\boldsymbol{j}-5\boldsymbol{k} + \lambda\left(5\boldsymbol{i}-4\boldsymbol{j}+2\boldsymbol{k}\right) = \left(-7+5\lambda\right)\boldsymbol{i}+\left(9-4\lambda\right)\boldsymbol{j}+\left(-5+2\lambda\right)\boldsymbol{k}$ $\boldsymbol{r}_2 = -6\boldsymbol{i}-5\boldsymbol{j}+2\boldsymbol{k} + \mu\left(9\boldsymbol{i}+6\boldsymbol{j}-3\boldsymbol{k}\right) = \left(-6+9\mu\right)\boldsymbol{i}+\left(-5+6\mu\right)\boldsymbol{j}+\left(2-3\mu\right)\boldsymbol{k}$ Solve $-7+5\lambda=-6+9\mu,\ 9-4\lambda=-5+6\mu$ and $-5+2\lambda=2-3\mu$ i.e. $\lambda=2$ and $\mu=1$ i.e. The smoke trails intersect at the point $(3,\ 1,\ -1)$
Specific behaviours
✓ correctly states the position vector of each rocket   ✓✓ correctly solves for λ and μ ✓ correctly identifies the point of intersection

(b) Determine the shortest distance of Rocket 1 from the smoke trail of Rocket 2, three minutes after firing. Give your answer to the nearest metre. (5 marks)

Solution



Let $\left|\overrightarrow{BC}\right|$ be the shortest distance between Rocket 1 and the smoke trail of Rocket 2, where $B(8,\ -3,\ 1)$ is the position of Rocket 1 three minutes after firing. $\overrightarrow{BC}$ is perpendicular to the smoke trail of Rocket 2. A is the point $(3,\ 1,\ -1)$.

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = -5\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k} + \lambda(9\boldsymbol{i} + 6\boldsymbol{j} - 3\boldsymbol{k})$

$\overrightarrow{BC}\,.\,9\boldsymbol{i} + 6\boldsymbol{j} - 3\boldsymbol{k} = 0$

i.e. $(-5\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k} + \lambda(9\boldsymbol{i} + 6\boldsymbol{j} - 3\boldsymbol{k}))\,.\,(9\boldsymbol{i} + 6\boldsymbol{j} - 3\boldsymbol{k}) = 0$

Solve using CAS:

$\lambda = \frac{5}{42}$ i.e. solve(dotP$([-5+9\lambda,\ 4+6\lambda,\ -2-3\lambda],[9,\ 6,\ -3]=0,\ \lambda))$

Shortest distance = 6574 m (to the nearest metre)

(i.e. norm$\left(\left[-5+\frac{45}{42},\ 4+\frac{30}{42},\ -2-\frac{15}{42}\right]\right) = 6.574$)

Specific behaviours

- ✓ correctly determines the position of Rocket 1, three minutes after firing
- ✓ recognises that $\overrightarrow{BC}$ is perpendicular to the smoke trail of Rocket 2
- ✓ correctly determines an expression for $\overrightarrow{BC}$
- ✓ correctly determines the value of λ
- ✓ determines the shortest distance

Question 16 **(7 marks)**

In a particular valley, only two types of plants grow: Blues and Reds. Today there are 130 hectares of Blue plants and 40 hectares of Red plants.

Due to a disease, that affects only Blue plants, the number of hectares of Blue plants, B , can be described by the differential equation $\frac{dB}{dt} = -0.15B$, where t is measured in years.

(a) In how many years will the number of hectares of Blue plants halve? (3 marks)

Solution
$B = 130e^{-0.15t}$ Thus for the half-life, $\frac{1}{2} = e^{-0.15t}$ $t = 4.621$ The number of hectares of Blue plants will halve in approximately 4.6 years
Specific behaviours
✓ correctly expresses B in terms of t ✓ states the correct half-life equation   ✓ calculates the final answer

(b) The number of hectares of Red plants, R , grows exponentially. Find the percentage growth rate in R if, after seven years, the total number of hectares of Blue and Red plants is the same as today. (4 marks)

Solution
$R = 40e^{kt}$ When $t = 7$: $130e^{-0.15\times7} + 40e^{7k} = 130 + 40$ Solve using CAS to find $k = 0.162$ The percentage growth rate of R is 16.2%
Specific behaviours
✓ correctly expresses R in terms of t ✓ states the correct equation   ✓ calculates the value for k ✓ states the percentage growth rate for R

Question 17 **(4 marks)**

(a) Write down the result (correct to five decimal places) of entering i^i (i to the power of i) into your CAS calculator. Comment on the answer you have recorded. (2 marks)

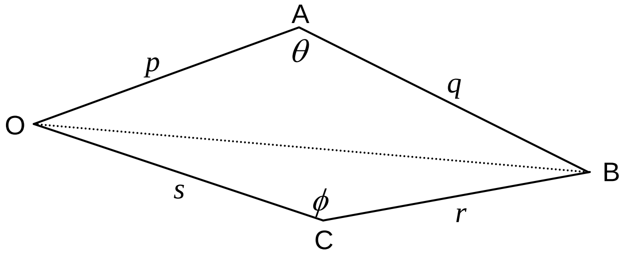
Solution
$i^i = 0.20788$ The answer is a real number
Specific behaviours
✓ correctly states the value of i^i ✓ notices that the answer is a real number

(b) Express your result for $z = i^i$ exactly in the form $z = a + bi$, where a and b are real. (2 marks)

Solution
$i^i = \left(e^{\frac{\pi}{2}i}\right)^i = e^{-\frac{\pi}{2}} = a$ Thus $z = e^{-\frac{\pi}{2}} + 0i$
Specific behaviours
✓ correctly converts i to its exponential form as $\left(e^{\frac{\pi}{2}i}\right)$ ✓ correctly determines the value of a

Question 18 **(10 marks)**

The quadrilateral OABC, shown below, has sides p, q, r and s, each of fixed length. The opposite angles θ and ϕ are variable.



(a) The cosine rule can be used in each of the triangles above to define the length of OB. Using these expressions for the length of OB and implicit differentiation, show that $\dfrac{d\phi}{d\theta} = \dfrac{pq\sin\theta}{rs\sin\phi}$. (3 marks)

Solution
$p^2 + q^2 - 2pq\cos\theta = r^2 + s^2 - 2rs\cos\phi$ Differentiate each side with respect to θ i.e. $2pq\sin\theta = 2rs\sin\phi\dfrac{d\phi}{d\theta}$ i.e. $\dfrac{d\phi}{d\theta} = \dfrac{pq\sin\theta}{rs\sin\phi}$
Specific behaviours
✓ correctly applies the cosine rule ✓✓differentiates correctly with respect to θ

(b) If A is the area of this quadrilateral, use calculus to show that $\frac{dA}{d\theta} = 0$ when θ and ϕ are supplementary (i.e. $\theta + \phi = 180°$). (7 marks)

Solution
$A = \frac{1}{2} pq\sin\theta + \frac{1}{2} rs\sin\phi$ Thus $\frac{dA}{d\theta} = \frac{1}{2} pq\cos\theta + \frac{1}{2} rs\cos\phi \frac{d\phi}{d\theta}$ i.e. $\frac{dA}{d\theta} = \frac{1}{2}\left(pq\cos\theta + rs\cos\phi \times \frac{pq\sin\theta}{rs\sin\phi}\right)$ i.e. $\frac{dA}{d\theta} = \frac{1}{2} pq\left(\cos\theta + \frac{\cos\phi\sin\theta}{\sin\phi}\right)$ i.e. $\frac{dA}{d\theta} = \frac{1}{2} pq\left(\frac{\cos\theta\sin\phi + \cos\phi\sin\theta}{\sin\phi}\right)$ i.e. $\frac{dA}{d\theta} = \frac{1}{2} pq\left(\frac{\sin(\theta + \phi)}{\sin\phi}\right)$ When $\theta + \phi = 180^\circ$, $\frac{dA}{d\theta} = \frac{1}{2} pq\left(\frac{\sin 180^\circ}{\sin\phi}\right)$ $= \frac{1}{2} pq\left(\frac{0}{\sin\phi}\right)$ $= 0$ Hence result
Specific behaviours
✓✓correctly expresses A in terms of the sides and angles of the quadrilateral ✓ differentiates A with respect to θ ✓ substitutes for $\frac{d\phi}{d\theta}$ from (a) ✓ simplifies correctly ✓ identifies and uses the trigonometric identity ✓ correctly shows $\frac{dA}{d\theta} = 0$ by substituting for $\sin(\theta + \phi)$

End of questions